Let's vote!

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September 7, 2022

1. A prose by Martin Niemöller

"First they came …" is the poetic form of a 1946 postwar confessional prose by the German Lutheran pastor Martin Niemöller (1892–1984). It is about the silence of German intellectuals and certain clergy—including, by his own admission, Niemöller himself—following the Nazis' rise to power and subsequent incremental purging of their chosen targets, group after group. It deals with themes of persecution, guilt, repentance, and personal responsibility. ---from wikipedia

First they came…

First they came for the Communists
And I did not speak out
Because I was not a Communist

Then they came for the Socialists
And I did not speak out
Because I was not a Socialist

Then they came for the trade unionists
And I did not speak out
Because I was not a trade unionist

Then they came for the Jews
And I did not speak out
Because I was not a Jew

Then they came for me
And there was no one left
To speak out for me

The prose indicates that those irrelative events (Nazis persecutes Communists, Socialist Jews, etc.), in the end, are closely related to the clergy's fate!

How could it happen? Here, we try to use data analysis to explain it.

2. Relevant or irrelevant

For simplicity, we use two dimensional vectors to represent certain social events.

For two given vectors $\bar{U} = (u_1, u_2)$ and $\bar{V} = (v_1, v_2)$. In a given society, the relation between two vectors can be measured by the angle between them, which can be calculated via the inner product of these two vectors. If the angle is 0^o or 180^o, we say two vectors are completely related (to be more precise: they are linearly related); if the angle is 90^o, we say they are completely irrelevant. Naturally, we call two vectors are closely related if the angle between them is close to 0^o or 180^o, and call two vectors are almost irrelevant if the angle between them is close to 90^o.

In fact, if we use θ to denote the angle between $\bar{U}$ and $\bar{V}$, we can use the inner product $< \bar{U}, \bar{V} >$ to compute

$$\cos\theta = \frac{< \bar{U}, \bar{V} >}{|\bar{U}||\bar{V}|},$$

where $|\bar{U}| = \sqrt{< \bar{U},\ \bar{U} >}$ is the length of $\bar{U}$, and $|\bar{V}| = \sqrt{< \bar{V},\ \bar{V} >}$ is the length of $\bar{V}$. In this note, we only talk about non-trivial vectors---those vectors with length larger than 0.

From Cauchy - Schwarz inequality, we know that for any two vectors $\bar{U}$ and $\bar{V}$,

$$\frac{| < \bar{U}, \bar{V} > |}{|\bar{U}||\bar{V}|} \leq 1$$

and the equality holds if and only if $\bar{U}$ // $\bar{V}$. So, if two vectors are completely related (thus they are parallel to each other), they are always completely related --- whatever the inner product you use to measure it.

We now re-exam the irrelevant relation.

We use the following example. Consider two vectors $\bar{F} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ and $\bar{W} = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$. Using the standard inner product on the plane: For $\bar{U} = (u_1, u_2)$ and $\bar{V} = (v_1, v_2)$,

$$< \bar{U}, \bar{V} >= u_1 v_1 + u_2 v_2$$

We know that

$$< \bar{F}, \bar{W} > = \frac{1}{2} - \frac{1}{2} = 0.$$

So $\bar{F}$ and $\overline{W}$ are completely irrelevant.

We will check the relation between $\bar{F}$ and $\bar{W}$ under a new inner product. For any two vectors $\bar{U} = (u_1, u_2)$ and $\bar{V} = (v_1, v_2)$, define

$$< \bar{U}, \bar{V} >_{new} = u_1 v_1 + 2 u_2 v_2$$

Using the new inner product, we calculate the length of vectors $\bar{F}$ and $\bar{W}$:

$$|\, \bar{F} \,|_{new} = \sqrt{< \bar{F}, \bar{F} >_{new}} = \frac{\sqrt{6}}{2}, \qquad |\, \bar{W} \,|_{new} = \sqrt{< \bar{W}, \bar{W} >_{new}} = \frac{\sqrt{6}}{2}.$$

Thus,

$$\frac{|< \bar{F}, \bar{W} >_{new}|}{|\, \bar{F} \,|_{new} |\, \bar{W} \,|_{new}} = \frac{4}{6} \times \left| \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} - 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \right) \right| = \frac{1}{3} \neq 0!$$

In layman’ s terms: from different viewpoint, two completely irrelevant event may NOT be completely irrelevant!

If we go a bit further, for any natural number k, we introduce a new inner product

$$< \bar{U}, \bar{V} >_k = k u_1 v_1 + u_2 v_2$$

for any two vectors $\bar{U} = (u_1, u_2)$ and $\bar{V} = (v_1, v_2)$.

Using the new inner product, we calculate the length of vectors $\bar{F}$ and $\bar{W}$:

$$|\, \bar{F} \,|_k = \sqrt{< \bar{F}, \bar{F} >_k} = \sqrt{\frac{k+1}{2}}, \qquad |\, \bar{W} \,|_k = \sqrt{< \bar{W}, \bar{W} >_k} = \sqrt{\frac{k+1}{2}}.$$

Thus,

$$\frac{|< \bar{F}, \bar{W} >_k|}{|\, \bar{F} \,|_k |\, \bar{W} \,|_k} = \frac{2}{k+1} \times \left| \left(k \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \right) \right| = \frac{k-1}{k+1}.$$

One can see that as k becomes large, the angle between $\bar{F}$ and $\bar{W}$ is getting close to 0^o or 180^o. So, two completely irrelevant events are getting more and more relevant due to the change of viewpoint (the increasing of parameter k)!

3. Let’s go voting

The politics looks like far away from mathematics and general science. After witnessing how the government dealing with various domestic and international issues in the past couple of years, are you convinced by the above argument to go voting?